Measurements of Embedding Impedance of Millimeter-Wave Diode Mounts

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***Abstract*—A method for measuring the embedding impedance of diode mounts is presented. The method is based on the measurement of reflection coefficient magnitude only. The reflection coefficient is measured as a function of diode bias (impedance). The embedding impedance can then be obtained in a simple way from the measured data. Results obtained on a coolable 60–90-GHz waveguide mixer are presented and discussed.**

I. Introduction

A PROBLEM that is often encountered at microwave- and millimeter-wave frequencies is that of experimentally determining load impedances for various devices mounted in actual circuits, such as mixer diodes, IMPATT diodes, etc. Difficulties arise because the terminals of the device are seldom physically accessible, i.e., the device is seen through an embedding network. A theoretical evaluation of the impedance parameters of this network is usually difficult due to the often complicated geometry of practical mounting structures.

In this paper, we will discuss a very simple reflectometer method for the determination of the load impedance seen from the device (diode) terminals in two-port circuits. In particular we will discuss measurements on waveguide-type millimeter-wave mixer mounts. The method may also be used for similar measurements on any two terminal device with a variable impedance element such as varactors, p-i-n diodes, etc.

Several papers dealing with the problem of measuring the elements of embedding networks have been published, e.g., [1]–[3]. The theoretical basis of the various techniques used will now be described in terms of the impedance matrix $[Z]$ of the embedding network. Let Z_d be the load (diode) impedance. The measured input impedance then is

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_d} \qquad (1)$$

where Z_{11}, Z_{22}, Z_{12}, and Z_{21} are the elements of the embedding network matrix $[Z]$. By varying Z_d and measuring Z_{in} it is possible to derive Z_{11}, Z_{22}, and $Z_{12} \cdot Z_{21}$, which can be used to calculate the load (mount) impedance Z_m seen by the device [1]

$$Z_m = Z_{22} - \frac{Z_{12} \cdot Z_{21}}{Z_{11}+Z_s} \qquad (2)$$

where Z_s is the impedance of the source.

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Z_d for a diode is often known from device physics and low-frequency measurements. It is also often a strong function of dc bias. By varying the dc bias one then has a variable Z_d.

The method to be described below differs from the one above in that it uses only reflection coefficient *magnitude* information, i.e., only $|\Gamma_{in}|^2$ is measured as a function of diode bias. (To measure Z_{in}, Γ_{in} is usually measured and Z_{in} calculated). It is then possible to determine Z_m but not Z_{11}, Z_{22}, and $Z_{12} \cdot Z_{21}$ separately. However, in some cases the latter quantities can still be determined by varying some other element in the network, e.g., the short-circuited position as will be discussed below.

The main advantage of the method is that very simple equipment can be used, e.g., a reflectometer setup will suffice. This should be contrasted with the expense of a network analyzer or the timeconsuming process of using standing wave techniques to obtain phase information. The method is convenient especially at millimeter wave frequencies where other equipment is presently unavailable.

In the following, we will limit our discussion to millimeter-wave mixer mounts. We think that modifications necessary for other applications should be fairly simple.

II. Embedding Impedance of Millimeter-Wave Mixers

To be able to predict mixer performance one must first of all know the embedding network at the signal, the image and the local oscillator frequency [4], [5]. These frequencies are separated by the intermediate frequency, which for mixers of this type can be quite high. For millimeter-wave mixers the usual approach has been to use impedance measurements made on scale-models at a much lower frequency than the actual signal frequency [2]. Even though valuable information can be gained in this way there are inherent difficulties with this technique. The shape of the contacting whisker is rather critical and cannot be accurately controlled and the diode chip is not negligibly small at the frequency of interest. It should, however, be pointed out that the technique described here can only be used in the fundamental mode frequency range and therefore embedding impedances at harmonic frequencies can in general not be determined.

A typical millimeter-wave mixer mount is shown in Fig. 1. The physical dimensions of the Schottky barrier diode are extremely small. The diameter of the diode junction is about 2 μm, while the width of the barrier is of the order

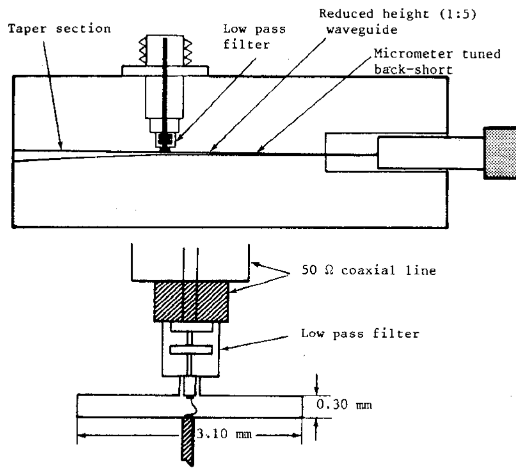

Fig. 1. A schematic diagram of a millimeter-wave (60–90-GHz) waveguide mixer.

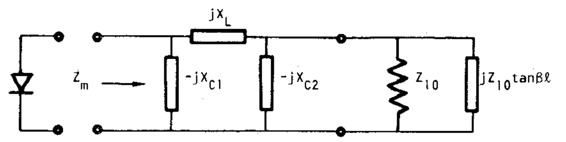

Fig. 2. Mixer mount equivalent circuit.

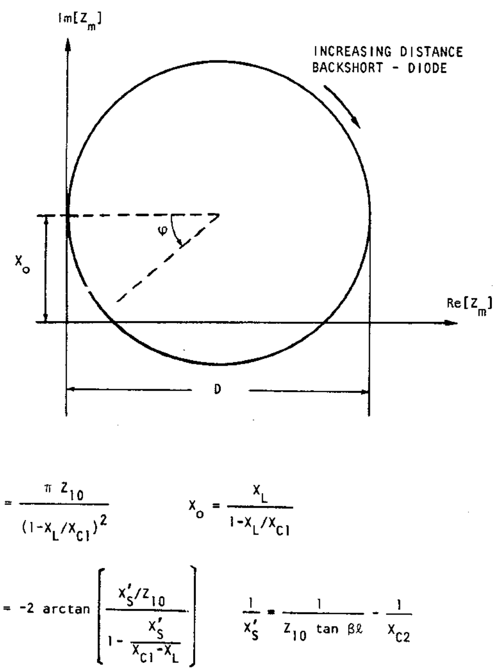

Fig. 3. The locus of the mount impedance Z_m.

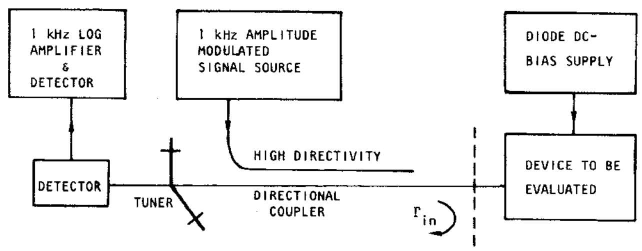

Fig. 4. Measurement setup.

0.1 μm. For a single-moded waveguide, the mixer mount, i.e., the embedding network, can be represented by the equivalent circuit shown in Fig. 2.

The elements of the π-network will in general be complex. They represent magnetic and electric energy (coupling to various cutoff modes) stored in various parts of the mount such as the inductive reactance of the whisker. If the network is lossy they will have a real part, if not they will be purely reactive. Z_{10} is the impedance of the dominant mode and $jZ_{10}\tan\beta l$ is the impedance of the variable short.

The mount impedance Z_m seen from the diode can easily be shown to describe a circle in the impedance plane when the position of the backshort is varied over $\lambda_g/2$. This fact has some important consequences when analyzing the embedding network of a mount and it will also serve as a test of the accuracy of a measurement series. In Fig. 3 we illustrate these properties of the mount and the relevant equations are given. We note that the diameter of the circle is proportional to the waveguide impedance Z_{10} and "magnified" by the factor $(1-X_L/X_{C1})^{-2}$. The reactive coordinate of the centre of the circle is proportional to the series elements X_L, and "magnified" by the factor $(1-X_L/X_{C1})^{-1}$. The angle φ becomes equal to $-2\beta l$ if the reactances of the parallel elements approach infinity.

III. Outline of the Method for Measuring Embedding Impedance

The embedding impedance for a fixed backshort setting may now be determined, knowing the diode impedance versus bias voltage and measuring the reflection coefficient *magnitude* $|\Gamma_{in}|$ of the mount. The measurement setup is illustrated in Fig. 4.

If the embedding network is lossless and we will assume this to be the case, we can write (see, e.g., [6])

$$|\Gamma_{in}|^2 = |\Gamma_m|^2 = \left|\frac{Z_m - Z_d^*}{Z_m + Z_d}\right|^2. \tag{3}$$

Note that the phases of Γ_{in} and Γ_m are not equal. For a lossy mount, (3) will be modified in a way which depends on where in the mount the losses occur. Z_d is the diode small signal impedance, which is a function of dc bias. It can be seen that if $Z_d^* = Z_m$ for some bias we will obtain a matched condition and $|\Gamma_m|^2 = 0$. Hence, if we measure $|\Gamma_m|^2 = 0$ we know that $Z_m = Z_d^*$.

For a Schottky barrier diode, the dc current through the diode when no RF power is present may be expressed as

$$i_{dc} = i_0(\exp^{(v/v_T)} - 1) \tag{4}$$

where

$$v = v_{dc} - i_{dc} \cdot R_s \qquad v_T = \frac{\eta k T}{e}.$$

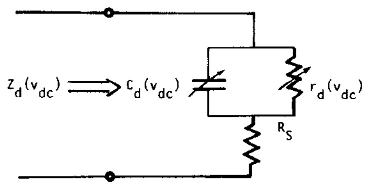

Fig. 5. The diode small signal equivalent circuit.

Here k is the Boltzmann constant, T the physical temperature of the diode, e is the charge of the electron, and η is an ideality factor (usually $\eta = 1.05 - 1.15$). The small signal (differential) resistance of the diode is

$$r_d = \left(\frac{di}{dv}\right)^{-1} = \frac{\eta k T}{e(i+i_0)}. \tag{5}$$

The small signal capacitance of the Schottky barrier diode is

$$C_d = \frac{C_0}{(1-v/\phi)^{\gamma}}. \tag{6}$$

Typically $\gamma = 0.45$ and $\phi = 0.95$ V for a GaAs Schottky barrier diode. From Fig. 5 we may now evaluate the load impedance $Z_d(v_{dc})$ as

$$Z_d(v_{dc}) = R_s + \frac{r_d}{1+(\omega C_d r_d)^2} - j\frac{r_d^2 \omega C_d}{1+(\omega C_d r_d)^2}. \tag{7}$$

With $Z_m = R_m + jX_m$ and $Z_d = R_d + jX_d$ we can rewrite (3) as

$$\left(R_m - \frac{1+|\Gamma_m|^2}{1-|\Gamma_m|^2}R_d\right)^2 + (X_m + X_d)^2 = R_d^2\left[\left(\frac{1+|\Gamma_m|^2}{1-|\Gamma_m|^2}\right)^2 - 1\right] \tag{8}$$

which is the equation of a circle in the impedance plane. The radius of the circle is

$$r_r = R_d\sqrt{\left(\frac{1+|\Gamma_m|^2}{1-|\Gamma_m|^2}\right)^2 - 1} \tag{9}$$

and the center coordinates are

$$r_0 = R_d \cdot \frac{1+|\Gamma_m|^2}{1-|\Gamma_m|^2} \qquad x_0 = -X_d. \tag{10}$$

Now if we know Z_d for a particular bias and the corresponding reflection coefficient we also know that the mount impedance Z_m will lie somewhere on the periphery of the circle given by (9), (10). To determine Z_m we obviously need more measurements at different diode bias voltages. These measurements will constrain Z_m to lie on other circles which will all intersect at the point Z_m.

This is illustrated in Fig. 6 where also a typical diode impedance locus is shown. Measurements made at three different voltages will suffice to determine Z_m. (Two measurements will give two possible solutions). In practice more than three measurements are useful since the circles will probably not intersect in one point due to measurements inaccuracies etc. In Fig. 7 a practical measurements series on a millimeter wave mixer is illustrated.

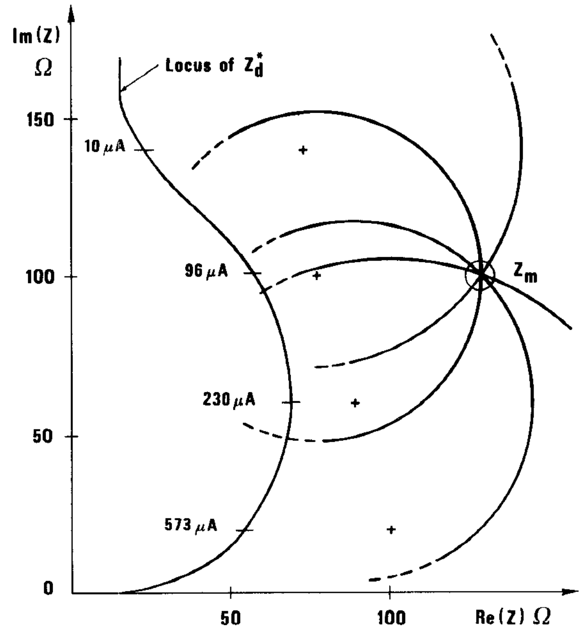

Fig. 6. Geometrical solution for Z_m with $R_s = 15$ Ω, $C_0 = 6.6$ fF, $\gamma = 0.45$, $\phi = 0.95$ V, and $f = 85$ GHz.

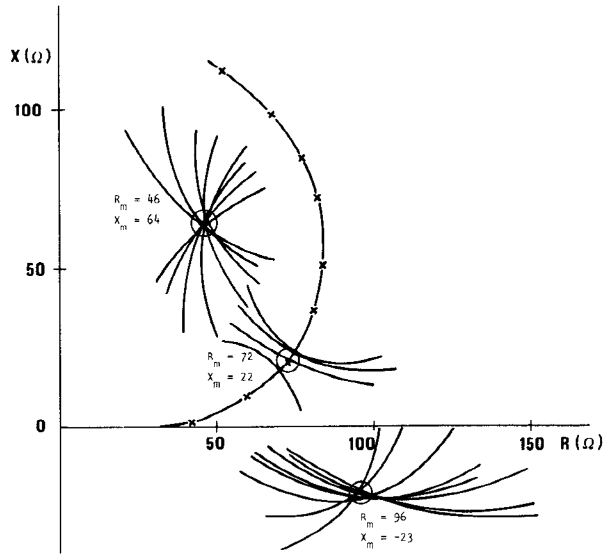

Fig. 7. Illustration of measurements made on a wafer type mixer at 87.8 GHz. The bias points used are indicated by crosses.

There are several possible variations of the above method, e.g., besides the mount impedance, also the series resistance and the zero bias capacitance of the diode can be determined [7], but the accuracy will be rather poor.

IV. The Minimum Method

It was shown above that measurements of $|\Gamma_m|^2$ at several different dc-current bias points will yield the information necessary to determine Z_m. In the "minimum" method we obtain this information by varying the bias until a minimum in $|\Gamma_m|^2$ is obtained (see Fig. 8).

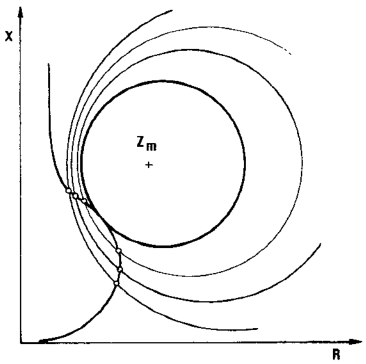

Fig. 8. Mismatch circles around the mount impedance Z_m. The circle corresponding to minimum $|\Gamma_m|^2$ will be tangent to the diode impedance locus.

The minimum $|\Gamma_m|^2$ and the corresponding diode impedance are then used to calculate the load impedance Z_m. The following relations can be derived by finding the minimum of $|\Gamma_m|^2$ versus the bias voltage v:

$$R_m = \frac{R_d}{\dfrac{1+|\Gamma_m|^2_{\min}}{1-|\Gamma_m|^2_{\min}} \mp \left[\left(\dfrac{1+|\Gamma_m|^2_{\min}}{1-|\Gamma_m|^2_{\min}}\right)^2 - 1\right]^{1/2} \cos\phi}$$

$$X_m = -X_d \pm R_m \left[\left(\frac{1+|\Gamma_m|^2_{\min}}{1-|\Gamma_m|^2_{\min}}\right)^2 - 1\right]^{1/2} \sin\phi$$

$$\phi = \arctan\left[-\frac{dR_d/dv}{dX_d/dv}\right]. \tag{11}$$

R_d, X_d are the diode resistance and reactance for the bias current corresponding to the minimum reflection coefficient $|\Gamma_m|^2_{\min}$. Choosing the right solution from the two possible solutions should cause no trouble since in most cases, it is obvious which one is the true one from other considerations [7].

We find this method very convenient in practice. The actual measurement is done quite rapidly and the analysis can be done using a pocket calculator.

In Fig. 11 in the next section we illustrate some measurement results obtained on a millimeter wave mixer for varying backshort position (for further details see [7]). The measured points fit a circle quite well and this circle is very nearly tangent to the "imaginary" axis. This is in accordance with the theory as discussed above and in more detail in [7]. The fact that the locus of the mount impedance is a circle, which is tangent to the imaginary axis offers a convenient way of checking the accuracy of the results. Systematic errors will show up in the measured impedances in such a way as that they will no longer lie on the periphery of a circle. For example, suppose that the diode capacitance is inaccurately known. To investigate this we calculated theoretical measurement series for a simple circuit with $X_{C1} = X_{C2} = \infty$ and $X_L = j85\ \Omega$. $|\Gamma_m|^2$ was calculated versus dc-bias to obtain $|\Gamma_m|^2_{\min}$ for different backshort positions. Using these values on $|\Gamma_m|^2_{\min}$ we then "applied" the minimum method with several different assumed values on C_0 as well as the correct one. The results are illustrated in Fig. 9. It can be seen that you will get quite visible deviations from a true circle with values on C_0 not differing more than 10–20 percent from the true value.

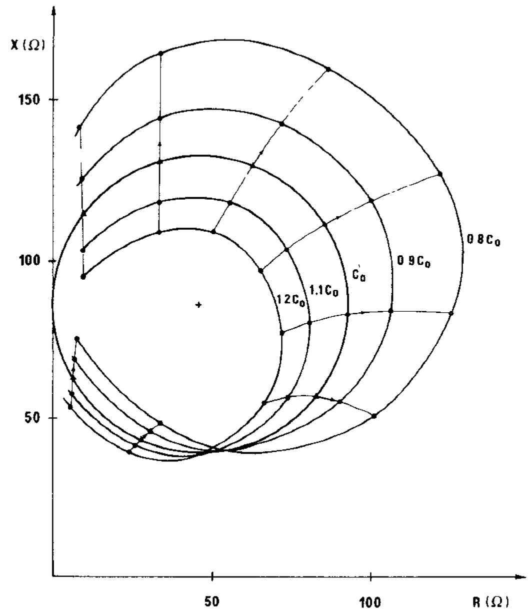

Fig. 9. Illustrating the effect of incorrect values on C_0.

It is also possible to investigate other sources of error in the same way, such as the presence of loss in the mount, measurement errors, etc. [7].

In conclusion we find the minimum method convenient, not overly sensitive to loss and small inaccuracies in the measurements. An inaccurate value on, e.g., zero bias capacitance will show up in the results in the form of distorted mount impedance circles.

V. Experimental Results Using the Minimum Method

To illustrate the usefulness of the minimum method we will discuss experiments performed on a 60–90-GHz mixer mount with a geometry as depicted in Fig. 1. This mixer was developed for cooled mixer experiments within the program for development of low noise receivers for the Onsala Space Observatory. The diode chip made from molecular beam epitaxi gallium arsenide was supplied by Dr. A. Y. Cho and Dr. M. V. Schneider at the Bell Telephone Laboratories, New Jersey (batch N277-81 in [8]). This diode chip has an extremely thin epitaxial layer with a low doping of 2×10^{17} cm^{-3} which should ensure low noise properties when operated at low temperatures. Room temperature data for the diode at 80 GHz are $R_S = 15\ \Omega$, $\eta = 1.14$ and the junction capacitance for backward bias voltages as shown in Fig. 10 were given to us by Dr. Schneider. A theoretical calculation of the depletion layer width (see, e.g., [10]) shows that it is equal to the thickness of the epitaxial layer at a forward bias of 0.5 V. Therefore, for bias voltages larger than 0.5 V the diode capacitance is predicted from (6) where, however, C_0 now

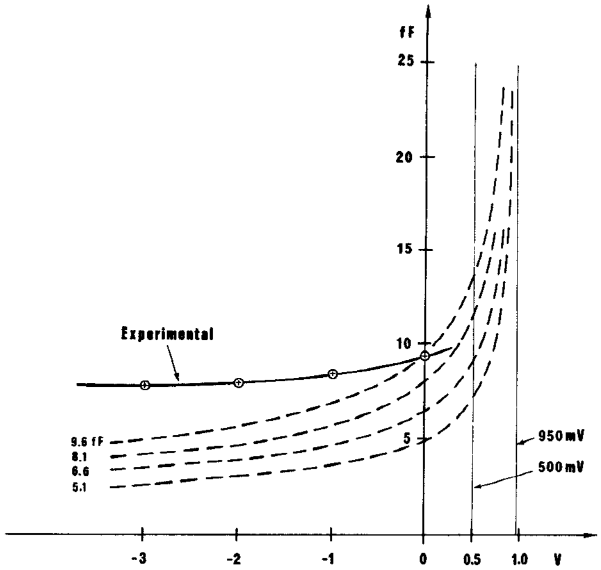

Fig. 10. Capacitance versus voltage for the diode used in the experiments compared with some capacitance versus voltage curves drawn using (6). ($\gamma=0.45, \phi=0.95$ V.)

will be lower than the measured zero bias capacitance. In Fig. 10 we have also depicted the capacitance versus the bias voltage as obtained theoretically for a diode on bulk GaAs, for various assumed junction capacitances at zero bias (C_0). The impedance characteristics of the mount were measured using the minimum $|\Gamma|^2$ method, discussed in Section IV. We used only forward bias voltages within the voltage range 600 mV$\lesssim V_{\text{bias}} \lesssim$880 mV and where possible, backward bias voltages for which the capacitance value was known (Fig. 10). For those forward bias voltages, we may consequently use (6) where we chose $\gamma=0.45$ and $\phi=0.95$ V.

Measurements of $|\Gamma_m|^2_{\min}$ were made for a great number of backshort positions and at several different frequencies. Different values for C_0 were tried in the analysis of the experimental data. The value of C_0 yielding mount impedance loci which approximated circles best was found to be 6.6 fF with an estimated error of about 1 fF.

Using minima found for bias voltages in the backward direction the maximum reactance of the circles could be determined. The results were, within the accuracy of the measurements, in excellent agreement with the assumption of a $C_0=6.6$ fF for the forward bias voltage regime.

In Fig. 11 measured impedance circles are presented for three different frequencies. As can be seen there is an obvious systematic change with frequency in the position X_0 and in the diameter D of the circles. For increasing frequency both D and X_0 increase. We have observed this effect in a number of other mounts as well. By measuring D, X_0 and also the angle φ we calculated X_{C1}, X_{C2}, and X_L [8]. The result is given in the Table I.

It is seen that in general the reactance X_{C2} is large. It should be compared to the waveguide impedance which is approximately 90 Ω. This is in agreement with the results of the Eisenhart and Khan model [10] where $X_{C2}=\infty$. It is interesting to compare these results with the idealized model. Using the Eisenhart and Khan theory we calculated the equivalent of X_{C1} and X_L. As can be seen from Table I the calculated X_L over the experimental was about 1.2, while the calculated X_{C1} (not shown in the table) was about two times larger than the experimental value. The agreement may be considered as rather good, in view of the difference between the actual mount and the model.

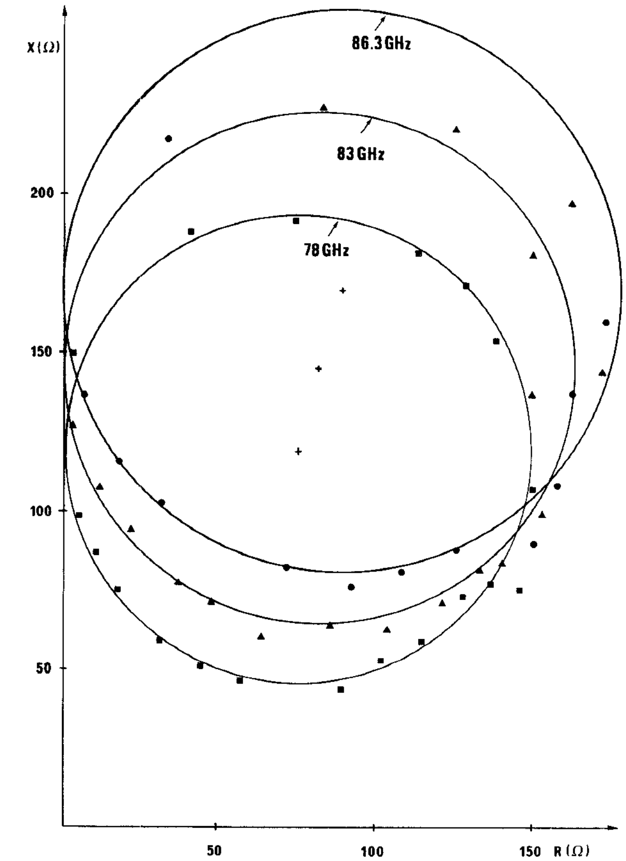

Fig. 11. Measured mount impedance (Z_m) for the mount of Fig. 1 using the minimum method.

TABLE I
ELEMENTS IN THE EQUIVALENT π-NETWORK OBTAINED FOR THE MOUNT OF FIG. 1.

f(GHz)	X_{C1}(Ω)	X_L(Ω)	X_{C2}(kΩ)	X_L(Ω) theory
78	470	95	−2.6 (ind.)	120
81	472	100	16.5	125
83	424	108	−2.1 (ind.)	129
86	408	120	2.5	135

Experiments have been performed on some other mounts with a larger waveguide height. We found these mounts to have very large diameter impedance circles. This is not only due to the higher impedance of the waveguide but also to the influence of the ratio X_L/X_{C1}.

In the evaluation of the experimental data one also has to consider that the series resistance of the diode is increased a few ohms [4] due to the skin effect. The value for the series resistance can in fact be determined to some accuracy if in the minimum method minima are found for low bias voltages (compare Fig. 5). However, for the minimum method and for a forward biased diode where $\text{Re}[Z_D]>R_S$, a few ohms error in the series resistance will only slightly effect the derived value for Z_m.

VI. Some Notes on the Experimental Procedure

The experimental setup was shown in Fig. 4. To get the best accuracy two tuners, one at each end of the directional coupler, are set as described in [6]. However, we have found that for most purposes it suffices to use only the tuner at the detector.

The power should be set at a low enough level to ensure linearity. For a Schottky mixer diode this will correspond to RF-junction voltages less than the thermal voltage ($\approx$25–30 mV). The RF-junction voltage due to the test signal can be estimated by noting the change in the dc through the diode when the test signal is applied. An increase of 26 percent is equivalent to a test signal voltage $V_P = v_T$.

The change is proportional to signal power ($V_P = V_T/\sqrt{2}$ will give a 13-percent increase) [7]. However, the easiest and safest way to check for linearity is of course to lower the power level until the measurement results are independent of the power level.

VII. Conclusions

A simple method to measure embedding impedance of diodes has been presented. The method is based on the measurement of reflection coefficient magnitude for varying diode bias. The embedding network is assumed to be lossless, which is, however, no serious limitation since for reasons of performance networks have low losses. The method is not overly sensitive to loss and small inaccuracies in the measurements, and the accuracy can be checked by, e.g., simulated measurement series.

Acknowledgment

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